A New Dispersion Model for Microstrip Line

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***Abstract*— This paper presents the phenomenological dispersion law and a new logistic dispersion model (LDM) for the microstrip line which has a root-mean-square (rms) accuracy of <1% and a maximum deviation of <2% for any W/h ratio, any permittivity, and at any operating frequency. The model is also applicable to the conductor of finite thickness. The eight existing dispersion models have also been compared against the experimental results and against the spectral-domain analysis (SDA) over a wide range of parameters.**

***Index Terms*—Dispersion, microstrip.**

I. Introduction

Several closed-form dispersion models for the microstrip line have been proposed in the literature either based upon some kind of mode-coupling phenomenon or based upon the curve fitting of dispersion data obtained from the rigorous theoretical formulations [1]–[8]. The coupling between the quasi-TEM mode and TM_o surface-wave mode suggests that with increasing frequency, more and more electromagnetic energy is confined to the dielectric region under the strip conductor. Thus, we can view concentration of the electric field lines below the conducting strip with increasing frequency as a phenomenon responsible for the dispersion in the microstrip line. This point of view suggests the phenomenological dispersion law and a logistic growth model for dispersion in the microstrip line.

Atwater [9] has noted that the available dispersion models have not been evaluated against a common full-wave program. He has expressed a need for such comparison in order to determine validity of various closed-form dispersion models over a broad range of parameters. The accuracy of dispersion models against the experimental results has also been examined [9]. However, separate informations regarding narrow and wide microstrip lines have not been presented. Thus, there is a need for a fresh evaluation of the existing dispersion models against the available experimental results and also against a common full-wave program.

In view of the above discussion, Section II of this paper attempts to establish a standard reference for comparison of the dispersion models. Section III presents a new logistic dispersion model (LDM) for the microstrip line for both zero and finite conductor thicknesses. Finally, Section IV compares nine dispersion models; namely, the models of Kirschning and Jansen (K–J) [8], Yamashita *et al.* (Yam.) [7], Kobayashi (Kob.) [3], modified Kobayashi (Mod. Kob.) [4], Hammerstad and Jensen (H–J) [6], Getsinger (Get.) [2], Pramanick and Bhartia (P–B) [5], Schneider (Sch.) [1], and the current LDM against the experimental results and a common spectral-domain analysis (SDA) program. One of this paper's reviewers had pointed out to the authors that the K–J model has been derived in a manner close to the present LDM [23].

II. Reference Standard for Comparision of Dispersion Models

For the systematic evaluation of the closed-form dispersion models, one of the full-wave-analysis algorithms should be taken as a standard reference. However, by collecting data from several numerical methods, Kuester and Chang [10] have shown that the total dispersion disagreement, i.e., $\epsilon_{\mathrm{eff}}(f) - \epsilon_{\mathrm{eff}}(0)$ at 8 GHz is as large as 25%. Even the best agreement between the variational method of Kowalski and Pregla [11] and the SDA of Capelle and Luypaert [12] is about 2% only. Yamashita and Atsuki [13] have found close agreement of their integral-equation method only with the results of Kowalski and Pregla [11]. Usually, the choice of the basis function influences the accuracy of the SDA. The basis function adopted by Jansen [14] provides dispersion results having a variation of 1%–2% against the experimental results.

Mirshekar and Davies have expanded the singular current density on the microstrip line in terms of Legendre polynomials [16], which resembles the longitudinal current density distribution of discrete modes of microstrip line determined by Capelle and Luypaert [12]. We have compared the results of the SDA of Mirshekhar–Davies (SDA MD) [16] with results of Kowalski and Pregla, and summarized by Hoffman [17], for $2.5 \leq \epsilon_r \leq 40$ and $0.1 \leq (W/h) \leq 5$. Both numerical techniques show root-mean-square (rms) deviation within approximately 1.7% for the range $2.5 \leq \epsilon_r \leq 40, 0.5 \leq (W/h) \leq 5$. The deviation increases with an increase in relative permittivity of the substrate. For the narrow line, i.e., $(W/h) = 0.1$, the maximum deviation is 2.3% at $\epsilon_r = 40$. This comparison has been done in the frequency range of 0–90 GHz. We have also compared the results of the SDA MD with the results of the SDA used by Kobayashi and Ando by using the closed-form expression for current density on the strip conductor [15]. Again, for the range $2 \leq \epsilon_r \leq 8$ and $1 \leq (W/h) \leq 2$, agreement is good. However, for the narrow line, i.e., $(W/h) = 0.4$, maximum deviation is in range of 3.72%–6.57%. The deviation increases with an increase in relative permittivity. This comparison has been done in the frequency range of 0–300 GHz, $0.4 \leq (W/h) \leq 2$, and $2 \leq \epsilon_r \leq 128$. We have also found that the dispersion results of the SDA MD agree within approximately a 2% range, with dispersion results obtained by the SDA of Capelle and Luypaert, the integral equation method of Yamashita and Atsuki, and the variational method of Kowlaski and Pregla.

We can next evaluate accuracy of the SDA MD against the experimental results which have been obtained from the graphical data of Edward and Owens [28] and Lee *et al.* [19]. The measurement results have been obtained in the frequency range of 0-18 GHz on sapphire, alumina, and barium tetratitanate $(\mathrm{BaTi_4O_9})$ substrates. The maximum deviation of the SDA MD against the experimental results is shown in the Table I. The rms and maximum deviations of SDA MD are 1.0% and 1.5%, respectively. The deviation is higher for $(W/h) < 0.5$ and at $\epsilon_r = 9.14$.

Keeping in view the above discussion, we can conclude that the SDA MD could be used as a theoretical standard reference for the comparison of all closed-form dispersion models. However, the standard itself has a deviation in the range of 1%–2% against the experimental results and other full-wave results. Thus, any model with a deviation of <1% could be treated as satisfactory and any model with deviation above 2% should be treated as not very satisfactory. For determination of accuracy of the closed-form dispersion models, a reference standard for the static value of effective relative permittivity $\epsilon_{\mathrm{eff}}(0)$ is also needed. Verma and Sadr [20] have compared $\epsilon_{\mathrm{eff}}(0)$ obtained from several models.

III. LDM

We can make the following statement on the phenomenological dispersion law for the microstrip line: the rate of increase of effective

Manuscript received August 13, 1996; revised February 26, 1998. The work of R. Kumar was supported by the Council for Scientific and Industrial Research (CSIR), India.

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Publisher Item Identifier S 0018-9480(98)05494-5.

TABLE I
MAX. % DEVIATION OF DISPERSION MODELS AGAINST EXPERIMENTAL RESULTS OF [18] AND [19]

ϵ_r	$\frac{W}{h}$	SDA [16]	K-J [8]	Yam. [7]	M. Kb. [4]	Kb. [3]	H-J [6]	Get. [2]	P-B [5]	Sch. [1]	LDM
	9.14	1.20	1.29	0.94	1.95	0.91	1.23	-1.85	-2.63	1.91	0.47
	4.98	0.69	-0.75	-1.22	-0.74	-1.3	1.16	-2.8	-3.07	8.12	-0.91
9.4⊥	1.259	0.47	-0.73	-1.92	0.47	1.94	0.88	0.58	2.09	4.40	-1.09
11.6∥	0.818	0.44	-0.70	-2.08	-0.46	1.55	-0.67	-0.72	2.01	2.88	1.40
	0.371	0.52	1.27	-0.71	1.45	2.92	0.97	1.22	3.87	2.31	0.89
	0.218	1.20	0.57	-1.2	0.79	1.84	0.84	0.93	3.11	0.95	0.59
	0.145	-0.73	0.49	-1.28	0.69	1.62	0.47	0.47	0.645	0.45	-0.71
	0.40	0.86	1.06	-1.41	1.15	2.57	0.95	0.96	3.19	2.01	1.30
10.15	0.90	1.02	1.15	-1.6	1.39	2.20	1.76	1.76	2.70	4.28	-1.56
	2.231	1.49	-1.14	-2.69	-1.07	1.45	-1.61	-3.02	-1.37	6.647	-1.31
37.0	1.0	0.94	-1.37	-3.83	-1.08	2.14	1.77	1.73	2.63	4.29	2.00

relative permittivity with frequency $\propto$ [Effective relative permittivity at the given frequency] $\times$ [Remaining fractional relative permittivity of the substrate].

This statement could be written as

$$\frac{d\epsilon_{\text{eff}}(f)}{df} = K\epsilon_{\text{eff}}(f)\left[\frac{\epsilon_r - \epsilon_{\text{eff}}(f)}{\epsilon_r}\right] \tag{1}$$

where K is the proportionality constant. The positive proportionality constant K is structure dependent. Its estimation will be examined later. The solution of (1) leads to the logistic dispersion equation

$$\epsilon_{\text{eff}}(f) = \frac{\epsilon_r}{1 + Me^{-Kf}}. \tag{2}$$

The dispersion expression should meet the following physical conditions given by Schneider [1]:

Initial conditions ($f \to 0$): (i) $\epsilon_{\text{eff}}(f) \to \epsilon_{\text{eff}}(0)$ and (ii) $(d\epsilon_{\text{eff}}(f)/df) \to 0$.

End Conditions ($f \to \infty$): (i) $\epsilon_{\text{eff}}(f) \to \epsilon_r$ and (ii) $(d\epsilon_{\text{eff}}(f)/df) \to 0$.

Using the initial condition, we get

$$M = \frac{\epsilon_r - \epsilon_{\text{eff}}(0)}{\epsilon_{\text{eff}}(0)}. \tag{3}$$

The operating frequency f in (2) could be normalized by the inflection frequency f_i. Thus, the logistic dispersion equation reduces to

$$\epsilon_{\text{eff}}(f) = \frac{\epsilon_r}{1 + Me^{-K(f/f_i)}}. \tag{4}$$

This expression meets the physical conditions of Schnieder, except the second part of the initial condition. The proportionality constant K is structure dependent, i.e., it depends upon the W/h ratio and relative permittivity ϵ_r of the substrate. It is also a function of frequency. Thus, it is difficult to determine a universal value of constant K. However, on computation of $\epsilon_{\text{eff}}(f)$ by the SDA MD and estimating the inflection frequency by (8), we can estimate the value of K by (5). The frequency-dependent behavior of K could be disregarded in the favor of the average value of K for each structure. At the inflection frequency, we get $K = \ln(M)$. However, this expression gives negative value for K, which is not acceptable. Instead of M, we can use the empirical factor $(2 - M)$ so that K becomes positive and its value comes into the range of the estimated value. Thus, the estimated empirical expression for K is given by

$$K = \ln\left[\frac{(3\epsilon_{\text{eff}}(0) - \epsilon_r)}{\epsilon_{\text{eff}}(0)}\right]. \tag{5}$$

The inflection frequency f_i could be determined from the coupling frequency $f_{k,\text{TM}}$ given by Kobayashi [3]

$$f_{k,\text{TM}} = v_o \frac{\tan^{-1}\left[\epsilon_r\sqrt{\dfrac{\epsilon_{\text{eff}}(0) - 1}{\varepsilon_r - \varepsilon_{\text{eff}}(0)}}\right]}{2\pi h\sqrt{\epsilon_r - \epsilon_{\text{eff}}(0)}} \tag{6}$$

where v_o and h are velocity of light in the free space and thickness of the substrate, respectively. The static relative effective permittivity $\epsilon_{\text{eff}}(0)$ could be obtained from either the accurate closed-form expressions of Hammerstad and Jensen [6] or by the variational method [21]. To take into account dependence of the inflection frequency on W/h ratio of the microstrip line, Kobayashi has suggested the following empirical relation for f_i:

$$f_i = \frac{f_{k,\text{TM}}}{\sqrt{3}\left(1 + \dfrac{W}{h}\right)}. \tag{7}$$

We have observed that f_i is also a function of relative permittivity of the substrate and the simple empirical expression (7) does not meet our requirement. Thus, we have modified the expression of inflection frequency to

$$f_i = \frac{f_{k,\text{TM}}}{\sqrt{3}\left(1 + B\,\dfrac{W}{h}\right)^A} \tag{8}$$

where the parameters A and B have been empirically determined by comparison of the present LDM against a large number of dispersion data obtained from the SDA MD [16]. Both the parameters are W/h dependent. However, A is also dependent upon ϵ_r. The curve-fit expressions for parameters A and B are summarized below.

Parameter A: For $1.05 \leq \epsilon_r \leq 10$

$$A = ax + b, x = \log_{10}\left(\frac{W}{h}\right). \tag{9}$$

1) (a) For $-1 \leq x \leq 0$, i.e., $(0.1 \leq (W/h) \leq 1)$

$$a = -0.1122\epsilon_r + 1.428 \quad b = 0.0649\epsilon_r + 0.3136. \tag{10}$$

2) For $0 < x \leq 0.7$, i.e., $(1 < (W/h) \leq 5$

$$a = -0.0927\epsilon_r + 0.9081 \quad b = 0.0648\epsilon_r + 0.3142. \tag{11}$$

3) For $0.7 < x \leq 1$, i.e., $((W/h > 5))$, $A = 0.95$.
For $10 < \epsilon_r \leq 20.0, x \geq -1$, i.e., $(W/h \geq 0.1)$, $A = 0.95$.
For $20.0 < \epsilon_r \leq 30.0$, $(W/h) \geq 0.6$.

$$A = \left(0.11639 \left(\frac{W}{h}\right)^{-0.1435} - 0.095\right)(\epsilon_r - 20) + 0.95. \quad (12)$$

For $30 < \epsilon_r \leq 200$, $(W/h) \geq 0.6$

$$A = 1.1639 \left(\frac{W}{h}\right)^{-0.1435}. \quad (13)$$

Parameter B: For $-1 \leq x < 0$, i.e., $(0.1 \leq (W/h) < 1)$

$$B = e^{-1.7047x}. \quad (14)$$

For $0 \leq x \leq 0.3$, i.e., $(1 \leq (W/h) \leq 2)$

$$B = 0.5(0.8 + e^{-1.7047x}). \quad (15)$$

For $x > 0.3$, i.e., $(W/h) > 2)$

$$B = 0.8. \quad (16)$$

Correction Factor in LDM: As mentioned above, the present dispersion model given by (4) does not meet the second part of Schneider's initial condition. Nonfulfillment of this condition generally results into a higher value for the effective relative permittivity calculated by the present model. Thus, a small correction term in the LDM is needed to improve the dispersion results in some range of microstrip parameters. However, as the dispersion expression is not variational in nature, a correction term may also degrade the effective relative permittivity in some other ranges. The form of the correction term could be obtained from the first derivative of (4) in which ϵ_r is replaced by $(\epsilon_r - \epsilon_{\text{eff}}(0))$ in order to avoid significant change in the nature of the LDM. Thus, the correction factor $\Delta\epsilon_r(f)$ could be written as

$$\begin{aligned} \Delta\epsilon_r(f) &= \frac{MK(\epsilon_r - \epsilon_{\text{eff}}(0))e^{-(K/(f/f_i))}}{1 + Me^{-K(f/f_i))^2}} \left(\frac{f}{f_i}\right), \qquad f \leq f_i \\ &= \frac{MK(\epsilon_r - \epsilon_{\text{eff}}(0))e^{-(K/(f/f_i))}}{1 + Me^{-K(f/f_i))^2}} \left(\frac{f}{f_i}\right), \qquad f > f_i. \end{aligned} \quad (17)$$

The final form of LDM becomes

$$\epsilon_{\text{eff}}(f) = \frac{\epsilon_r}{(1 + Me^{-K(f/f_i)})} - E\Delta\varepsilon_r(f) \quad (18)$$

where in the range of $1 < \epsilon_r \leq 20$

$$E = 0, \quad \text{for} \quad 0.1 \leq \frac{W}{h} < 5 \quad \text{and} \quad E = 1, \quad \text{for} \quad \frac{W}{h} \geq 5 \quad (19)$$

and in the range of $20.0 < \epsilon_r \leq 200$

$$E = 1, \quad \text{for} \quad 0.1 \leq \frac{W}{h} < 1 \quad \text{and} \quad E = 0 \quad \text{for,} \quad \frac{W}{h} \geq 1. \quad (20)$$

Finite Conductor Thickness: The LDM has been obtained from the phenomenological dispersion law and, therefore, it is also applicable to the microstrip line with a finite strip-conductor thickness t. Thickness of the strip conductor could be accounted for in the calculation of static $\epsilon_{\text{eff}}(t, 0)$ [22]. The $\epsilon_{\text{eff}}(t, 0)$ could be used in the above expressions in place of $\epsilon_{\text{eff}}(0)$. The W/h ratio in the above-mentioned expressions could be replaced by the $W_{eq}(t, f)/h$ ratio, which is a function of the conductor thickness t and the operating frequency f

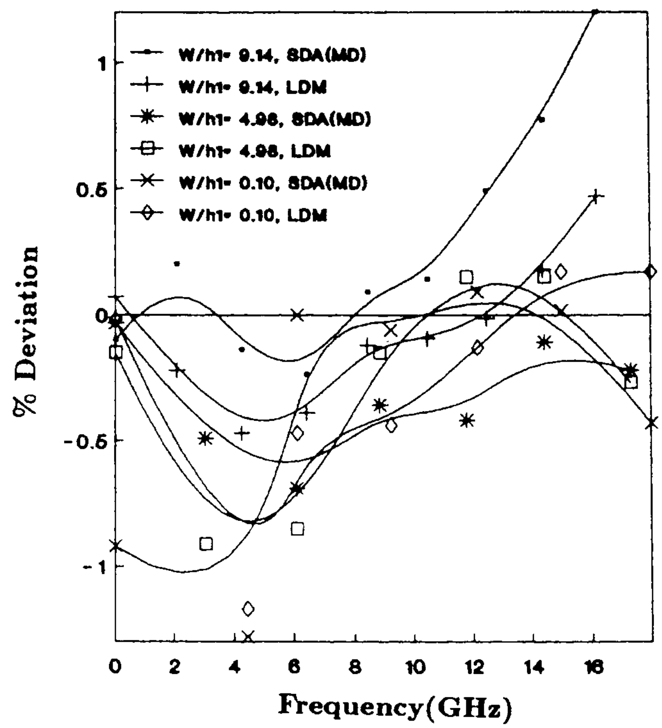

Fig. 1. % deviation of the SDA MD and LDM against experimental results $(\epsilon = 9.4 \perp, 11.6 \parallel)$.

$$\epsilon_{\text{eff}}(t, 0) = \epsilon_{\text{eff}}(t = 0, f = 0) - \frac{(\epsilon_r - 1)\dfrac{t}{h}}{4.6\sqrt{\dfrac{W}{h}}} \quad (21)$$

$$W_{eq}(t, f) = W + \frac{W_{eq}(t, 0) - W}{1 + \left(\dfrac{f}{f_o}\right)^2} \quad (22)$$

$$f_o = \frac{C_o}{2W_{eq}(t, 0)\sqrt{\epsilon_{\text{eff}}(t, 0)}} \quad (23)$$

where C_o is the velocity of light. The static $W_{eq}(t, 0)$ and $\epsilon_{\text{eff}}(t, 0)$ can be obtained from Garg and Behl [22]. $W_{eq}(t, f)$ has been adopted from Owens [24].

IV. Accuracy of Closed-Form Dispersion Models

We have compared the eight dispersion models against the available experimental results in the 2–18-GHz frequency range on sapphire and alumina substrates [18] and on the high permittivity $(\epsilon_r = 37)$ barium tetratitanate substrate [19]. The dispersion models have been compared against the SDA MD [14] over a wide range of parameters. The maximum deviation of each dispersion model on three substrates for several linewidths is given in the Table I. Fig. 1 compares the LDM and results of the SDA MD against the experimental results [18], [19]. The LDM has a better agreement with experimental results as compared to the SDA MD. In the whole range, the rms deviation of the SDA MD, K–J, Mod. Kob., and LDM are within 1%. The maximum error of the SDA MD and LDM is 1.4%, whereas maximum deviation of K–J and Mod. Kob. is 1.3% and 1.95%, respectively. All other models have a higher deviation. For the wide line, i.e., $(W/h) = 9.14$, the LDM has a maximum deviation of 0.47%, whereas the K–J and Mod. Kob. models have maximum deviation 1.3% and 1.95%, respectively. Kirschning and Jansen have

TABLE II
DEVIATION OF DISPERSION MODELS AGAINST SDA MD ($h = 0.02$ cm, $0 \leq f \leq 100$ GHz)

ϵ_r	Maximum Deviation									
	$\frac{W}{h}$	K-J	Yam.	Mod. Kob.	Kob.	H-J	Get.	P-B	Sch.	LDM
	0.10	2.49	2.02	2.53	2.54	2.35	1.74	2.76	2.24	1.47
2.2	1.00	0.35	-0.08	0.49	0.72	0.50	-2.92	-0.47	2.23	-1.24
	10.0	0.28	0.37	0.53	-0.97	-0.71	-2.44	-2.69	4.26	-1.01
	0.10	2.50	1.80	2.72	2.75	1.60	1.60	6.02	-3.40	-1.28
9.8	1.00	0.18	-2.37	0.58	1.92	-4.36	-4.79	2.28	4.16	-1.38
	10.0	0.5	0.73	0.93	-1.54	1.77	-2.59	-3.20	8.43	-1.37
	0.10	2.46	-1.76	2.50	2.97	-2.66	2.30	6.94	-7.49	-1.65
20.0	1.00	0.16	-2.79	0.58	2.50	-4.60	-4.50	2.80	4.55	1.70
	10.0	0.61	0.83	1.00	-1.70	2.30	-2.50	-3.20	8.93	-1.39
	RMS Deviation									
2.20	$0.1 \sim 10$	0.737	0.751	0.845	0.951	0.853	1.382	1.301	1.829	0.851
9.80	$0.1 \sim 10$	0.683	1.017	0.861	1.444	1.564	1.866	2.253	4.393	0.921
20.00	$0.1 \sim 10$	0.716	0.139	0.877	1.482	1.563	1.982	1.609	4.186	1.115

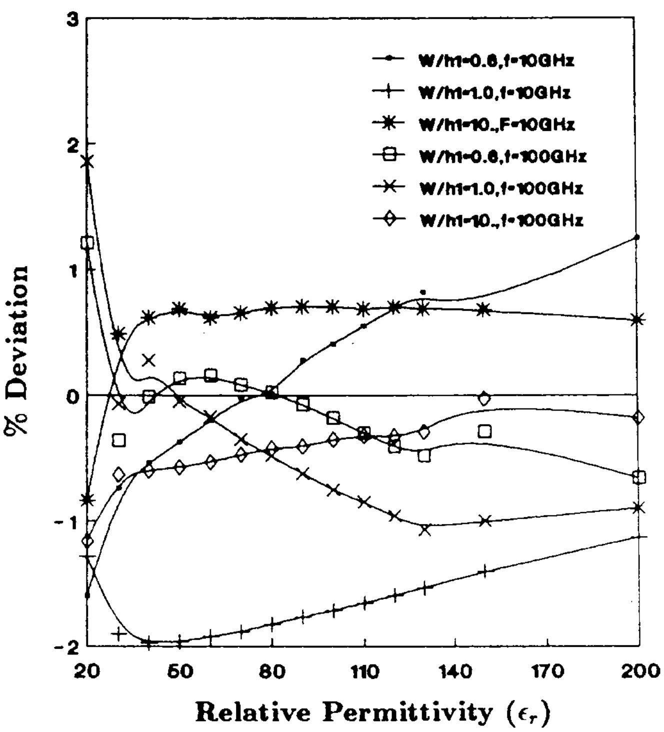

Fig. 2. % deviation of the LDM against the SDA MD.

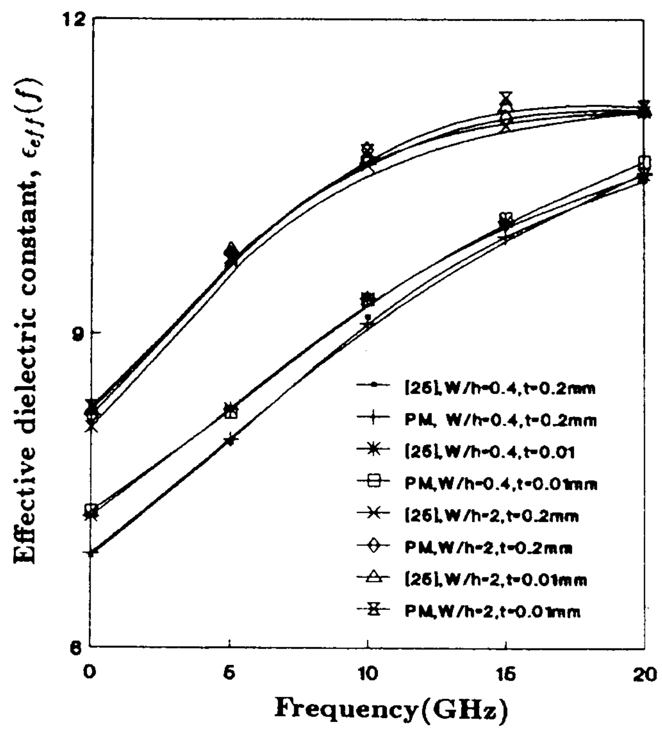

Fig. 3. Comparison of dispersion results of the present model LDM, i.e., for finite strip thickness against [25].

also reported the accuracy of their model around 1%–2% against the experimental results.

Table II shows the maximum and rms deviations in the dispersion models against the SDA MD. It is obvious that for $(W/h) = 0.1$, the LDM has better performance. In the whole range, the LDM has a maximum deviation of 1.70%, whereas K–J and Mod. Kob. have deviations of 2.76%. The rest of the models have higher deviations. Accuracy of the LDM has been further tested in the range $20 < \epsilon_r \leq 200, 0.6 \leq (W/h) \leq 20$. The results shown in Fig. 2 indicate that for most of the cases, deviation in the LDM is within 1%.

The SDA MD is not valid for the microstrip line with finite conductor thickness. Therefore, the LDM for $0 \leq t \leq 0.2$ mm on $\epsilon_r = 11.7$ has been tested against the variational conformal mapping [25] up to $f \cdot h \leq 6$ GHz $\cdot$ cm [25]. Fig. 3 shows that the agreement of results is good, i.e., within 1.2%.

V. CONCLUSION

In this paper, we have presented the LDM, which has rms deviation within 1%, maximum deviation ≤1.4% against the experimental results for $\epsilon_r \leq 37$, and frequency up to 20 GHz. For any permittivity at any operating frequency and any W/h ratio, the LDM has rms deviation of about 1% and a maximum deviation <2%. The LDM is also applicable to the microstrip line with finite conductor thickness.

It has an accuracy comparable to the existing dispersion models over a much wider range of parameters. The present model is open in nature, i.e., it can be adopted by any design engineer to achieve better accuracy in the model to match his measurement results for any specific substrate. The designer has to simply recalculate the A and B parameters against the experimental results and curve fit the data by linear or power regression, as presented in this paper. Moreover, the LDM is very simple and fast for computer-aided design (CAD) application and, with some modification, it could be adopted to model the dispersion in other planar transmission lines. The LDM is also suitable for effective presentation in classroom teaching.

Acknowledgment

The authors express their gratitude to the reviewers for citing [23] and improvement in the paper's discussion of the proportionality constant. They are grateful to the reviewer who insisted upon the incorporation of the conductor thickness in the model. The authors are also thankful to Prof. E. K. Sharma for careful reading of the manuscript.

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New Empirical Unified Dispersion Model for Shielded-, Suspended-, and Composite-Substrate Microstrip Line for Microwave and mm-Wave Applications

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***Abstract*—By introducing the concept of "virtual relative permittivity," this paper reports several closed-form dispersion models for a multilayered shielded/unshielded microstrip line over $1 < \epsilon_r \leq 20$, $0.1 \leq (w/h) \leq 10$, $(h_3/h) \geq 2$ in the frequency range up to 4 GHz · cm. The maximum deviation of the one model against the results of the spectral-domain analysis (SDA) is limited to 3%, while for the other three models, the maximum deviation is <2% and the root-mean-square (rms) deviation is <0.8%. This paper also reports improvement in the closed-form model of March for the determination of $\epsilon_{\mathrm{eff}}(0)$ of the shielded microstrip line.**

***Index Terms*—Dispersion, multilayer microstrip.**

I. Introduction

The closed-form models are normally preferred by the designers due to their simplicity and ease in use. However, closed-form expressions for dispersion in the microstrip line on a composite/suspended substrate with and without a top shield are not available in the open literature. Jansen [1] has concluded that the dispersion modeling becomes extremely involved if the physical parameters of microstrip-like lines exceed four. Using the concept of the single-layer reduction (SLR) formulation, Verma and Hassani developed a unified dispersion model [2] for the shielded/unshielded multilayer microstrip line. However, this model degrades over the wider range of parameters. Replacement of the composite-substrate microstrip line by an equivalent permittivity of a single substrate has also been suggested by Finlay *et al.* [3]. However, no analytical method has been suggested by them for its determination.

Manuscript received October 2, 1996; revised February 26, 1998. The work of R. Kumar was supported by CSIR, India.

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Publisher Item Identifier S 0018-9480(98)05496-9.